
sPION
Delft 2018
ASML
Stichting Physica
University of Amsterdam
NWO
Nederlandse Organisatie voor Wetenschappelijk Onderzoek
Nikhef
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PION 2018 - Delft

Introduction

Dear PION-competitor,

After a good cup of coffee or tea, an informative lecture and a filling lunch, it is time for the reason why you are here; the Olympiad! We would like to present the problems of PION 2018! With many thanks to the professors we have made a set of problems that combine many parts of the physical world surrounding you. These exercises will be more challenging than any exam you have ever made. We would like to wish you all the best with the Olympiad, show us why you should go to Zagreb this year!

Good luck!

Bart Warmerdam, Kim Bosch, Heike Smedes, Boyd Voet, Dirk van Bolhuis, Rixt Bosveld
PION commissie 2018

Rules and general information:

- Every problem should be made on a separate sheet.
- There will be 9 problems.
- Not every problem is worth the same number of points. The maximum of points that you can get per problem can be seen on the next page. With a total of 90 points.
- You have 3 hours to work on the problems.
- Write your team name and the name of the problem on each sheet.
- Only BINAS is allowed to use as reference.
- It is forbidden to communicate with any one but your teammates and the PION committee.
- It is forbidden to use a graphic calculator with more features than a TI-84 or equivalent calculator.

On the cover

The image on the cover is a laser show combined with an atom, which is the logo of the VVTP. Source: https://www.showandstage.de/Bundle-Laserworld-CS-2000RGB-MKII-Pangolin-Quickshow-10m-ILDA-Kabel-Case .

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The 90 points are distributed as follows:

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Tweedejaarsstudenten kunnen een jaar gratis lid worden. Alle andere studenten krijgen een flinke korting op de contributie.

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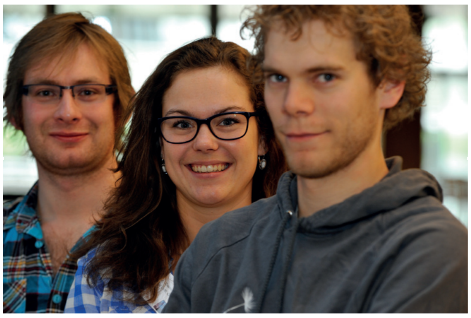

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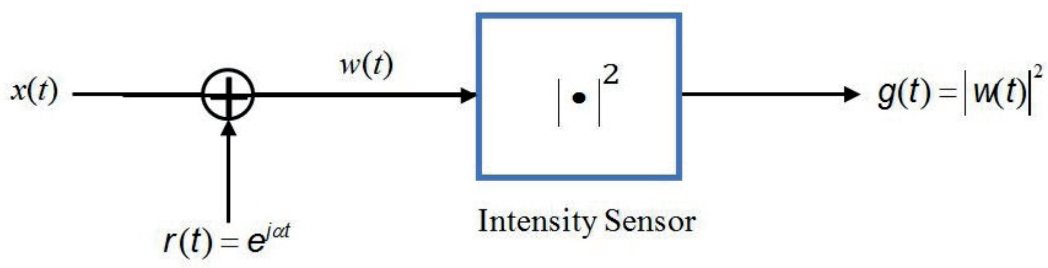
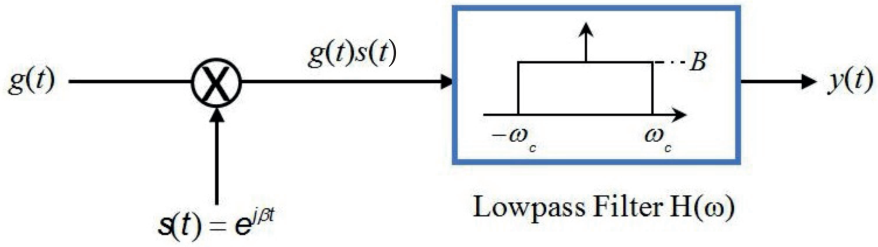
Holography

The system shown below is a one-dimensional representation of the imaging process known as holography. To understand how it works, we investigate the system for a bandlimited input signal x(t).§
The recording aspect of holography is modeled as follows:



Holographic Recording

The reconstruction aspect of holography is modeled as follows:



Holographic Reconstruction

Assume that the Fourier transform of x(t) is:

$$X(\omega)=\begin{cases} 1+\delta(\omega) & 0\le|\omega|\le 1 \\ 0 & elsewhere \end{cases}$$

where the unit impulse function δ(u) is defined by:

$$x(v)=\int_{-\infty}^{+\infty} x(v-u)\delta(u)du$$

Problem a. (2 pts) Determine and make a labeled sketch of the input signal x(t).

Problem b. (1 pt) Determine and make a labeled sketch of $W(\omega)$, the Fourier transform of $w(t) = x(t) + r(t)$.

Problem c. (2 pts) Recording: Determine and make a labeled sketch of $G(\omega)$, the Fourier transform of $g(t) = |w(t)|^2$. Your answer need only be expressed in terms of $X(\omega)$ and impulse functions.

Problem d. (3 pts) Reconstruction: For $\alpha = 6$, determine the value for the cutoff frequency ω_c and the amplitude B of the lowpass filter and β so that the output $y(t) = x(t)$ for all t.

Problem e. (2 pts) In principle, every camera with "film" is capable of recording an image. What makes the model of holographic imaging, as described above, special?

§In order to be compatible with standard formula sheets, such as the one attached, the process represented above is in the time domain (t) instead of the normal, holographic spatial domain (x). Nevertheless, it is a one-dimensional model of the two-dimensional, holographic imaging process. Holography consists of two steps: 1) Recording the hologram with a sensor such as film and 2) reconstructing the image by illumination of the hologram. The sensor is sensitive only to the intensity (that is, the magnitude squared) of the incoming wave (light) that falls upon it. Thus, in order to record both magnitude and phase of the original image on film, a reference plane wave is added prior to making the film recording. The image is reconstructed by illuminating the hologram with a second plane wave.

Note in the following formulas: $j^2 = -1$

FORMULA SHEET

	Continuous time	Discrete time
Fourier Series	$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\frac{2\pi}{T}t}$ $a_k = \frac{1}{T}\int_0^T x(t) e^{-jk\frac{2\pi}{T}t} dt$	$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\frac{2\pi}{N}n}$ $a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-jk\frac{2\pi}{N}n}$
Fourier Transform	$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega$ $X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$	$x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(\Omega) e^{j\Omega n} d\Omega$ $X(\Omega) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\Omega n}$
Laplace Transform and z-Transform	$X(s) = \int_{-\infty}^{+\infty} x(t) e^{-st} dt$	$X(z) = \sum_{n=-\infty}^{+\infty} x[n] z^{-n}$
Parseval's Theorem for aperiodic functions	$\int_{-\infty}^{+\infty} \lvert x(t)\rvert^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \lvert X(\omega)\rvert^2 d\omega$	$\sum_{n=-\infty}^{+\infty} \lvert x[n]\rvert^2 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \lvert X(\Omega)\rvert^2 d\Omega$
Parseval's Theorem for periodic functions	$\frac{1}{T}\int_0^T \lvert x(t)\rvert^2 dt = \sum_{k=-\infty}^{+\infty} \lvert a_k\rvert^2$	$\frac{1}{N}\sum_{n=0}^{N-1} \lvert x[n]\rvert^2 = \sum_{k=0}^{N-1} \lvert a_k\rvert^2$

Prof. Dr. I.T. Young

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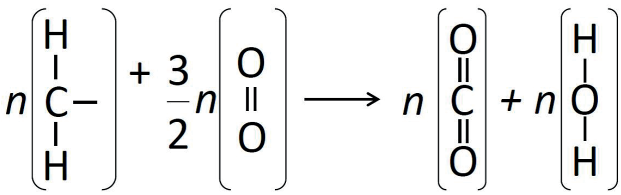
The K.J-U Space Program

President K.J-U at P. has the ambition to send a manned spaceship from Earth to the nearest star, Alpha Centaurus (=Proxima Centaurus). All astronauts will be males. The spaceship will be powered using fossil fuels.

source: SanDiegoUnionTribuneCreators.com

Problem a. (3 pts) Estimate the minimum kinetic energy that needs to be given to the spaceship and show that this energy is much larger than the minimum escape energy from Earth.

Problem b. (3 pts) Estimate the heat of combustion (in J/kg) of a fossil fuel (=hydrocarbon).
Hint (1): The combustion reaction for a long linear hydrocarbon with chain length *n* can be approximated by



Hint (2): Use binding energies as given in Binas.

(If you have not been able to answer (b), then use the heat of combustion for Butane as given in Binas.)

Problem c. (1 pt) If the empty mass of the spaceship is m_{ss} and the mass of the needed fossil fuel is M_{FF}, then estimate the ratio M_{FF}/m_{ss}.

Problem d. (2 pts) After having seen the above analysis, President Kim Jong-un at P. changes plans and decides that the spaceship should be powered by a nuclear fusion process, in which hydrogen is converted into helium. When the minimum needed mass of hydrogen is M_{H2}, then estimate the ratio M_{H2}/m_{ss}.

Prof. Dr. C. R. Kleijn

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Particle Creation in Strong Fields

Classically particles are indestructible, but quantum-mechanically two particles can annihilate into pure energy. Similarly pure energy can create particles. A strong enough electric field can spontaneously generate electron-positron pairs.
The fact that this can happen, is already visible in standard non-relativistic single particle quantum mechanics. The Hamiltonian of a non-relativistic charged particle in an electrostatic field is

$$\mathbf{H} = \frac{1}{2m}\mathbf{p}^2 + e\Phi$$

For a constant electric field in the z-direction, the electrostatic potential is

$$\Phi(x, y, z) = E_z z$$

Problem a. (1 pt). What is funny/uncomfortable about the Hamiltonian of a particle in a constant electric field? Hint: think about the groundstate.

This Hamiltonian ignores something very physical. To create particles one must add energy in according to Einstein's rest energy $\varepsilon_{rest} = mc^2$, or more generally

$$\mathcal{E} = \sqrt{\mathbf{p}^2 c^2 + m^2 c^4}.$$

Since the Schrödinger equation treats momentum and energy different, it is not compatible with special relativity and we must generalize it. The simplest such generalization is the Klein-Gordon equation

$$\left((\mathbf{H} - e\Phi)^2 - (\mathbf{p} - e\mathbf{A})^2 c^2 - m^2 c^4\right)\phi = 0$$

where for completeness we have shown how the vector potential **A** also enters. This equation does not exactly apply to electrons (it ignores the fact that they are fermions and have spin), but is sufficient for our purposes. We now set **A** = **0** again. The associated differential equation a relativistic particle in the electrostatic potential as above (in the second equation) is

$$\left((i\hbar\frac{\partial}{\partial t} - eE_z z)^2 + \hbar^2 c^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) - m^2 c^4\right)\phi = 0$$

Problem b. (2 pts) Substitute in the ansatz

$$\phi(x, y, z, t) = e^{-i\mathcal{E}t/\hbar} e^{ip_x x/\hbar + ip_y y/\hbar} \phi_{\mathcal{E}, p_x, p_y}(z)$$

The resulting equation looks like an ordinary time-independent Schrödinger equation:

$$\left(\hbar^2 c^2 \frac{\partial^2}{\partial z^2} - V_{eff}(z)\right)\phi_{\mathcal{E}, p_x, p_y}(z) = 0$$

What is the effective potential $V_{eff}(z)$ as a function of z?

Problem c. (2 pts) What is the shape of this potential assuming $\varepsilon, p_x, p_y, E_z, m$ are constant? Solve for the two values z_{min} and z_{max} where $V_{eff} = 0$.

Problem d. (1 pt) Note that the differential equation is now second order in $\partial/\partial z$. As a result the equation has two solutions. One for the particle and one for the antiparticle. Show that the antiparticle with opposite charge $e \rightarrow -e$ and opposite "energy" $\varepsilon \rightarrow -\varepsilon$ obeys the same equation. Show also that the antiparticle $e \rightarrow -e$ in the mirror world $z \rightarrow -z$ is also a solution to the same potential.

You might wonder how to think about a solution with "negative energy". A intuitive way to resolve this is that the solution $\varphi_\varepsilon \sim e^{iEt+ipz}$ can be either thought of as a solution with negative energy moving in the same direction as the particle, or as a solution with positive energy moving in the other direction. This is precisely reflected in the fact that the antiparticle solution is the complex conjugate solution after also changing $e \rightarrow -e$. The exact solution to the relativistic wave equation (the 7th equation) is hard, but we can easily solve it in the WKB approximation.

Problem e. (1 pt) Substitute the ansatz

$$\phi_{\mathcal{E},p_x,p_y}(z) = A_{\mathcal{E},p_x,p_y} e^{\frac{i}{\hbar}\int^z dz' p_z(z')}$$

into the realistic wave equation. Now make the approximation that $\partial/\partial z\ (p_z(z)) << p_z(z)$ and any other term. Write down the equation for $p_z(z)$.

Problem f. (1 pt) For $p_z(z)$ is real, we have an oscillatory solution

$$e^{\frac{i}{\hbar}\int^z p(z')dz'}$$

i.e. a real wave. Given fixed values of p_x, p_x, ε, m argue that for $z << z_{min}$ and $z >> z_{max}$, this is always so for the shape of the effective potential.

The oscillatory solution with $p_z(z) > 0$ for $z << z_{min}$ describes (is the wavefunction of) a particle moving to the right. Similarly the oscillatory solution for $p_z(z) < 0$ is a particle moving to the left.
The oscillatory solution with $p_z(z) < 0$ for $z >> z_{max}$ can be either thought of as a particle moving to the left or an antiparticle moving to the right.

Problem g. (2 pts) For $p_z(z)$ is imaginary, we have an exponentially decaying/growing solution and we can have tunneling solution from $z << z_{min}$ and $z >> z_{max}$. The tunneling amplitude is given by the change in the amplitude accross the barrier. This is precisely

$$T = \exp(-\frac{1}{\hbar}\int_{z_{min}}^{z_{max}} dz|p_z(z)|)$$

Compute T. A useful integral is

$$\int_{-1}^{1} dx\sqrt{1-x^2} = \frac{\pi}{2}.$$

Classically there is no tunneling of course, and an incoming particle moving to the right will be completely reflected. Quantum mechanically there is tunneling with amplitude T and probability T^2 and without proof we will state that the tunneling amplitude matches an incoming particle wavefunction with $p_z(z) > 0$ at $z < z_{min}$ to a solution with $p_z(z) < 0$ at $z > z_{max}$. The only reasonable solution is to think of this as an antiparticle also moving to the right. Thus in the process a particle-antiparticle pair must have been created.

Further reading

1. A. Hansen, F. Ravndal, Phys.Scr. 23 (1981) 1033.
2. S. P. Kim, D. N. Page, Phys.Rev. D65 (2002) 105002.
3. H. Kleinert, R Ruffini, S.-S. Xue, Phys.Rev. D78 (2008) 025011.

Prof. Dr. K. E. Schalm

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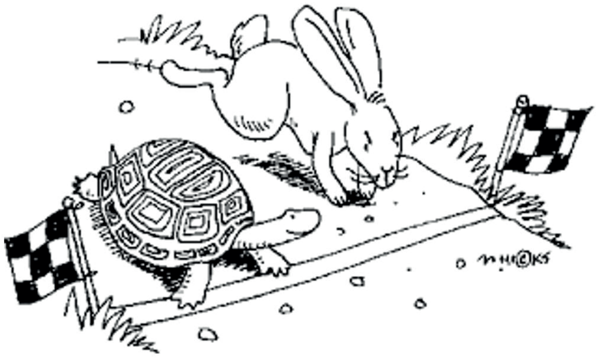
Turtle versus Hare

Lorentz transformations

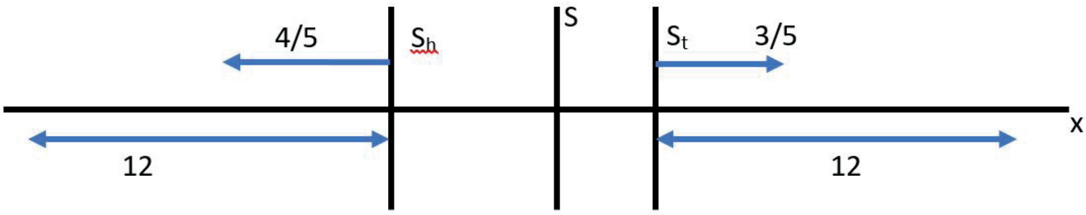
$$ct' = \gamma\left(ct - \frac{V}{c}x\right)$$

$$x' = \gamma\left(x - \frac{V}{c}ct\right)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}$$



Turtle has been thinking. She has been losing from hare during all previous races. Obviously, says turtle to herself, hare will always be faster, so I need to change the game fundamentally.
When the next annual race approaches, turtle proposes to hare the following to make the race more interesting. Rather than racing the same track, each of them will race his/her own track that is an exact copy of the track of the other. The difference is that turtle will run the track in an inertial frame of reference that moves with respect to the referee in the positive x-direction with exactly the speed that turtle can run. Similarly, hare will run its track that is in an inertial frame that moves in the -x-direction at the same speed that hare can run. So, their tracks move along with their respective inertial frames.



The rules are the following.

- At t=0, the origin of all three inertial frames (S of the referee, S_t in which turtle runs, S_h in which hare runs) coincides and the clocks are all three set to zero.
- Turtle runs at a velocity of 3/5 of the speed of sound and hare runs at 4/5 of the speed of sound. *Both velocities are measured in their own inertial frame.*
- S_t moves with 3/5 of the speed of sound away from S and S_h does so at 4/5 of the speed of sound.
- Both turtle and hare run a similar track: a distance of length L which is equivalent to 12 seconds for sound traveling this distance. Then they instantly run back to their own origin. There they hit a button and a loud sound beep is made.
- The referee in S (in the origin) marks the time when she hears the beep.
- If the turtle's beep is registered first, the turtle wins. If the hare's beep is registered first, the hare wins.

Problem a. (3 pts) Determine the winner of the contest.

Many years later, the descendants of turtle and hare are still racing each other. But by training and natural selection, they both run much faster: turtle at 3/5 of the speed of light and hare 4/5 of that. They repeat the above race, but sound is replaced by light and the sound beep by a light flash.

Problem b. (3 pts) Determine the winner of the new contest.

Problem c. (4 pts) Show that if turtle trains a little extra such that she runs at

$$\frac{v}{c} = \frac{\sqrt{5}-1}{2}$$

hare can never win, no matter how fast (or slow) he will run.

Prof. Dr. R. F. Mudde

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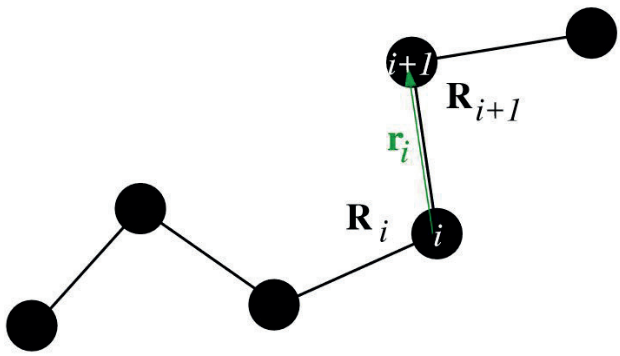
Elasticity of Polymers

Polymers can be modelled as chains consisting of *beads*, which are connected by rods. In this problem, we consider a simple model of a polymer in which each rod has a fixed length a, and successive rods are *freely jointed*, which means that the energy of the polymer does not depend on the relative orientation of successive rods.

The beads have positions that we denote as $\mathbf{R}_i$, and the rods are described by the vectors $\mathbf{r}_i = \mathbf{R}_{i+1} - \mathbf{R}_i$. There are N+1 beads and therefore N rods; the index i runs from 0 to N.



The *conformation* of a polymer is given by the vectors $\mathbf{r}_i$. The probability density for finding a polymer in a particular conformation is a constant, that is, each conformation is equally probable, provided the distance between successive beads is a.

Problem a. (2 pts) The end-to-end vector $\mathbf{R}$ of the chain is defined as $\mathbf{R} = \mathbf{R}_N - \mathbf{R}_0$. Argue that

$$\langle \mathbf{r}_i \cdot \mathbf{r}_j \rangle = a^2 \delta_{ij},$$

where $\delta i j$ is the Kronecker delta-function. Use this result to show that

$$\langle \mathbf{R}^2 \rangle = N a^2.$$

(It suffices to argue the validity of the second equation using heuristic arguments.)

The probability density for a conformation of the freely jointed chain is given as

$$P_N(\mathbf{R}_0, \ldots, \mathbf{R}_N) = \left(\frac{1}{4\pi a^2}\right)^N \prod_{i=0}^{N-1} \delta(|\mathbf{r}_i| - a),$$

where $\delta(x)$ is the Dirac delta function. Note that this indeed expresses what was noted above: there is no dependence on the orientations of the $\mathbf{r}_i$; only the bondlengths are fixed.
The end-to-end vector $\mathbf{R}$ is defined as $\mathbf{R} = \mathbf{R}_N - \mathbf{R}_0$. The probability density for this end-to-end vector is called $P^E_N(\mathbf{R})$. This probability density is found by fixing the first bead at $\mathbf{R}_0 = \mathbf{0}$ and the last one at some position $\mathbf{R}_N = \mathbf{R}$, and then integrating the above probability density for the conformations over the positions of beads 1,2,...,N−1.
Evaluating this probability seems an impossible task, but it turns out possible by taking the Fourier transform of $P^E_N(\mathbf{R})$:

$$\tilde{P}^E_N(\mathbf{k}) = \int P^E_N(\mathbf{R}) e^{i\mathbf{k}\cdot\mathbf{R}} d^3 R,$$

followed by some approximation.

Problem b. (3 pts) Show that the Fourier transform is given by

$$\tilde{P}_N^{\mathrm{E}}(\mathbf{k}) = \left(\frac{\sin(ka)}{ka}\right)^N.$$

Hint 1: Change the integration variables from $\mathbf{R}_i$ to $\mathbf{r}_i$ and show that the Jacobian for this transformation is 1.
Hint 2: You can use the fact that

$$\int e^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}}\delta(r-a)d^3r = 4\pi a^2\frac{\sin(ka)}{ka}.$$

If you have time left, you can show that this equation is true. Show that $\tilde{P}_N^{\mathrm{E}}(\mathbf{k})$ has its maximum at $\mathbf{k}=\mathbf{0}$. Use Taylor expansion of sin(ka)/ka around $\mathbf{k}=\mathbf{0}$ to show that

$$\ln\tilde{P}_N^{\mathrm{E}}(\mathbf{k}) \approx -N\frac{k^2a^2}{6}.$$

Problem c. (3 pts) Carry out the inverse Fourier transform of this last form of $\tilde{P}_N^{\mathrm{E}}(\mathbf{k})$ in order to show that

$$P_N^{\mathrm{E}}(\mathbf{R}) = \left(\frac{3}{2\pi Na^2}\right)^{3/2}\exp\left(-\frac{3R^2}{2Na^2}\right).$$

Hint: $\int_0^\infty e^{-ax^2}dx = \sqrt{\pi/a}/2$.

In statistical mechanics, the probability density for the states of a system is proportional to the Boltzmann factor $e^{-\beta E}$, where E is the energy of the state and $\beta=1/(k_BT)$, k_B is Boltzmann's constant and T is the absolute temperature. We therefore identify the probability density for the conformations with the Boltzmannfactor (even though it does not depend on the temperature T). From the calculation, it should be clear that, for fixed end points, you have integrated this probability density over all the positions of intermediate points. In statistical mechanics, this sum (or integral in the case of continuum variables) of the Boltzmann factors over all possibilities, is the *partition function*. In our case, the possibilities are all the conformations with given $\mathbf{R}=\mathbf{R}_N-\mathbf{R}_0$.

Problem d. (2 pts) Argue that the free energy of a polymer with fixed endpoints is given by $-k_BT\ln P^{\mathrm{E}}_N(\mathbf{R})$. Show that stretching the end points of the polymer, this acts as a spring with a spring constant κ given by

$$\kappa = \frac{3k_{\mathrm{B}}T}{Na^2}.$$

Next time you see a rubber band, think about this calculation :-)

Dr. J. Thijssen

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Newton Rings

In 1664, the polymath Robert Hooke, in his book Micrographia, described a phenomena that he could not understand. Here an excerpt:

> Take two small pieces of ground and polish Looking-glass-plate, each about the bigness of a shilling, take these two dry, and with your fore-fingers and thumbs press them very hard and close together, and you shall find, that when they approach each other very near, there will appear several Irises or coloured Lines, ..., and you may very easily change any of the Colours of any part of the interposed body, by pressing the Plates closer and harder together, or leaving them more lax; that is, a part which appeared coloured with a red, may be presently ting'd with a yellow, blew, green, purple, or the like, by altering the appropinquation of the terminating Plates.

Figure 1 :Left: Robert Hooke(1635-1703); Right: Isaac Newton (1643-1727)

Problem a. (0.5 pts) Make a drawing for the experiment described by Hooke.

Problem b. (0.5 pts) Which physical phenomenon gives rise to these lines?

These lines or rings observed by Robert Hooke were later studied by Sir Isaac Newton in 1717. A Scientific battle started between Hooke and Newton about who discover/work on their topics the first. Time gave credit to Newton and from that moment on, they were called Newton's rings. Rings occurs when two surfaces - a sphere surface and an adjacent flat surface - are illuminated by light.
Let consider two surfaces made of a thin flat glass plate and a spherical glass bowl of radius R. The air gap in between the two surface is showed in Figure 2. The light is monochromatic with wavelength in vacuum λ_0 and is coming from above. You are looking from above.

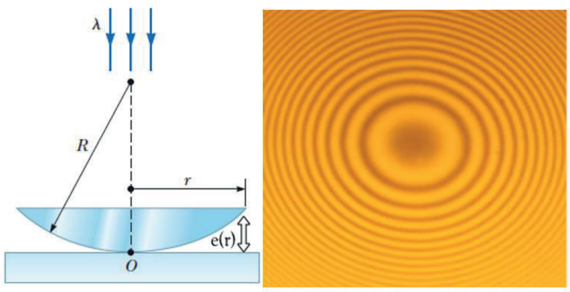


Figure 2: Optical set-up to observe the Newton's Rings shown in the right by someone looking from above

Problem c. (1.5 pts) Explain with a drawing and with words why do you observe these rings (Figure 2 -right). Why there is not light in the center of the picture? Draw what would you see if you were looking from below.

Let's try to model mathematically what is happening.

Problem d. (1 pt) Calculate the distance *e* between the two surfaces as function of *r*.

Problem e. (1 pt) We assume that $R >> r$. Show that *e(r)* can be approximated to:

$$e(r) = \frac{r^2}{2R}$$

Problem f. (2 pts) Show that the bright rings will appears for the radii $r_{b,m}$ expressed by:

$$r_{b,m}^2 = (m - 1/2)\lambda R$$

where $\lambda=\lambda_0/n_{air}$ is the wavelength in air. Define *m*.

Problem g. (0.5 pts) What are the radii $r_{d,m}$ for the dark rings?

Now let's focus on some calculation from the formula you have found. In his book "Optiek", Newton wrote:

> The same Experiment I repeated with another double convex Object-glass ground on both sides to one and the same Sphere. Its Focus was distant from it 168-1/2 Inches, and therefore the Diameter of that Sphere was 184 Inches. This Glass being laid upon the same plain Glass, the Diameter of the fifth of the dark Rings, when the black Spot in their Center appear'd plainly without pressing the Glasses, was by the measure of the Compasses upon the upper Glass 121/600 Parts of an Inch, and by consequence between the Glasses it was 1222/6000: For the upper Glass was 1/8 of an Inch thick, and my Eye was distant from it 8 Inches. And a third proportional to half this from the Diameter of the Sphere is 5/88850 Parts of an Inch. This is therefore the Thickness of the Air at this Ring, and a fifth Part thereof, viz. the 1/88850th Part of an Inch is the Thickness thereof at the first of the Rings, as above.

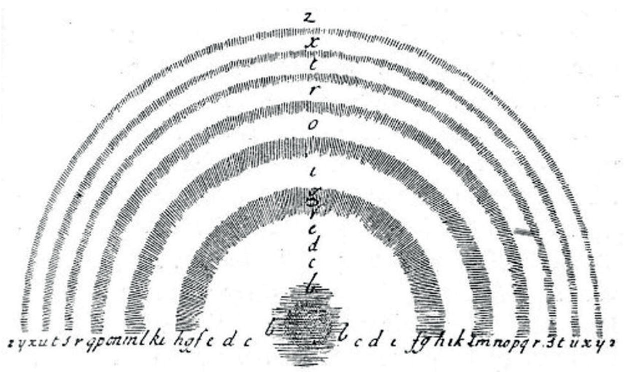


Figure 3:The ring's as Newton Described them in his book "Opticks".

Problem h. (0.5 pts) Does the last measurement (1/88850th of an inch for the air gap at the first dark ring) correspond to your findings, assuming that Newton was using a yellow light of λ_0=570 nm.

Problem i. (0.5 pts) Further, Newton wrote: “The Squares of the Diameters of these Rings made by any prismatick colour were in arithmetical Progression, as in the fifth Observation.” Was he correct?

Problem j. (1 pt) Find a simple relation for the distance between two consecutive dark rings ($r_{d,m+1}$-$r_{d,m}$) versus λ, R and m.

Problem k. (0.5 pts) Newton also illuminate apparatus with light coming out of a prism and wrote down where the colors end up as shown in his description in Figure 4. Explain why he sees “rainbow” of colors.

Problem l. (1 pt) We consider now a lens with radius R=5.0 m and of diameter Φ=20 mm. We use the yellow emission of a sodium lamp (λ=589 nm). How many dark rings do we observe?

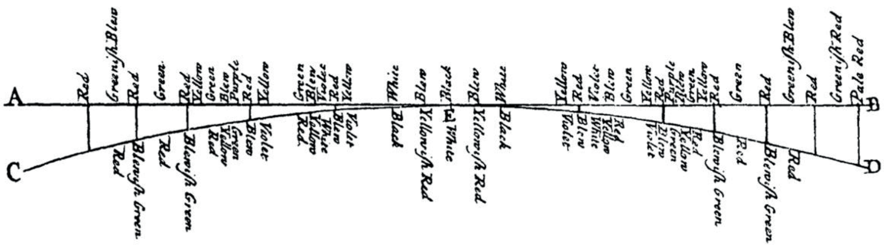


Figure 4: The color has seen by Newton and described in his book “Opticks”.

We place the apparatus inside water, so the gap is now filled with water. We observe 45 dark rings. Extract the index of refraction of water from these measurements.
Problem m. (0.5 pts) Give a possible other use (or application) of the Newton rings.

Problem n. (0.5 pts) What would happen if the glass plate is replaced by a plane mirror?

Problem o. (0.5 pts) In the Figure 5 we observe the following “rings” when measuring a lens in a Newton’s-rings apparatus. Could you explain what may have caused them?

Figure 5: Measurement of pattern from a lens in a Newton’s-rings apparatus.

Dr. A. J. L. Adam

Nikhef is het Nationaal instituut voor subatomaire fysica. Het instituut doet onderzoek naar de elementaire bouwstenen van ons universum, hun onderlinge krachten en de structuur van ruimte en tijd. Wetenschappers en technici werken samen om antwoorden op de grote natuurkundige vragen van deze tijd te vinden. Een groot deel van het onderzoek van Nikhef vindt plaats bij de deeltjesversneller op CERN, de 'Large Hadron Collider'. Daarnaast is Nikhef actief in de astrodeeltjesfysica waarbij interacties van hoogenergetisch deeltjes vanuit de kosmos in bijvoorbeeld de atmosfeer of zeewater worden waargenomen. Ook het onderzoek naar zwaartekrachtsgolven en het bestaan van donkere materie staat in een hoog vaandel.

Nikhef is een samenwerkingsverband op het gebied van (astro)deeltjesfysica tussen de Nederlandse Organisatie voor Wetenschappelijk Onderzoek (NWO) en vijf universiteiten: de Radboud Universiteit, de Rijksuniversiteit Groningen, de Universiteit van Amsterdam, de Universiteit Utrecht en de Vrije Universiteit Amsterdam.

PION 2018 - Delft

Correcting Quantum Errors

On the the major challenges in the research field of quantum computing is to deal with imperfect quantum operations and decoherence of qubits. In this exercise, we will take a look at a method that can be used to mitigate these problems, called quantum error correction (QEC).
When working with multiple qubits, unitary gate operations are often represented in circuit diagrams (examples later). The most common unitary operations are given below. These elements are called quantum gates.

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

The basis in which these gates are expressed is the $\{|0\rangle, |1\rangle\}$ basis for single qubit gates and the $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ basis for the two qubit gates.

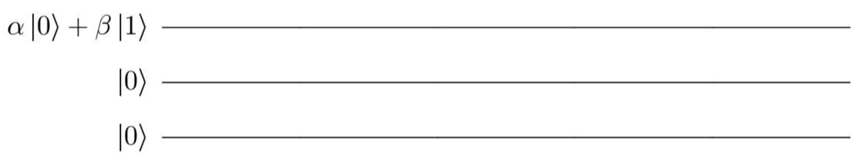
Encoding a single qubit. A simple quantum error correction encoding scheme, called the repetition code, encodes a single logical qubit in the state of three physical qubits. For example:

$$|\psi\rangle = \alpha |0\rangle_L + \beta |1\rangle_L = \alpha |000\rangle + \beta |111\rangle \tag{1}$$

where the subscript L refers to the logical qubit. This state can be created by applying some of the gates given above. The starting state is given by:

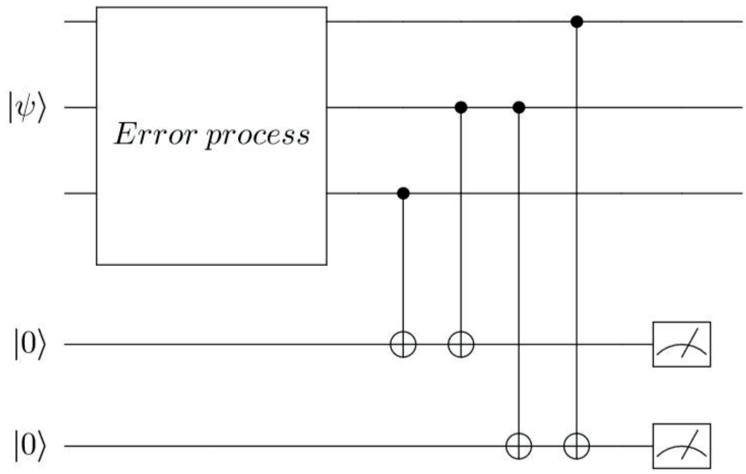
$$|\psi_0\rangle = (\alpha |0\rangle + \beta |1\rangle) \otimes |0\rangle \otimes |0\rangle \tag{2}$$
$$= \alpha |000\rangle + \beta |100\rangle \tag{3}$$

Problem a. (1 pt) Create the state given in equation 1, starting $|\psi_0\rangle$. Make use of the circuit elements given above and draw them in the circuit diagram below (draw the circuit on your **answer sheet**). In this diagram, each line represents one qubit. Time goes from left to right.



Problem b. (2 pts) Is this state entangled? Why/why not?
(note that you do not need the outcome of this part to proceed to the next part)

Protecting quantum information. In the following scheme, a specific implementation of the repetition code is shown that allows for correcting bit flip errors (e.g. a X error on one of the qubits).



The top three lines represent the logical qubit, as starting state we will take the state that was encoded in the previous exercise. The two bottom qubits represent so-called ancilla qubits. These are helper qubits that at the end are measured by projection in the $\{|0\rangle, |1\rangle\}$ basis.

Problem c. (4 pts) Explain how a single bit flip error (an unintentional X operation) is detected using this circuit, i.e. how the outcome of the measurement of the ancilla qubits allows you to infer on which of the three qubits that encode $|\psi\rangle$ the bit flip error occurred. *Hint: consider the symmetry of the wavefunction*.

Problem d. (1 pt) In case you detect a bit flip error on one of the qubits, how could you correct it? Make a truth table of all possible measurement outcomes and propose for each outcome the correction step for the most likely error

Problem e. (2 pts) How would you change this circuit to protect against a dephasing error (an unintentional Z operation) on a single qubit?

Prof. Dr. L. M. K. Vandersypen en MSc S. Phillips

Electrostatic Lenses in the MAPPER Electron Lithography Machine

The company MAPPER Lithography BV that has arisen from the TU Delft is developing a machine with which patterns for advanced chips can be written into the electron-sensitive material directly from the computer. One of the advantages in comparison to the usage of a "mask", is that every chip on a wafer can get an own pattern which can be used as a security code. To enable the machine to be quick enough, the writing happens with thousands of bundles in parallel. These bundles are focused with very small electrostatic electron lenses. To make a sufficient amount of bundles the lenses have a typical diameter of 150 micron. The most simple electrostatic lens consists of a field that ends on an plate with a hole. Electrons stay relatively close to the axis and therefore the field may be described in a series-expansion around the axis.

Problem a. (4 pts) Given that the potential is described by $\Phi(0,z)$ and the field in the z-direction by $E_z(0,z)$, show that $\Phi(r,z)$, expressed in a series-expansion which shows the lens effect as well as the spherical aberration, can be written as:

$$\Phi(r,z) = \Phi(0,z) - \frac{r^2}{4}\frac{\partial^2\Phi(0,z)}{\partial z^2} + \frac{r^4}{64}\frac{\partial^4\Phi(0,z)}{\partial z^4} - \cdots$$

Also calculate the series-expansion of $E_z(r,z)$ and $E_r(r,z)$.

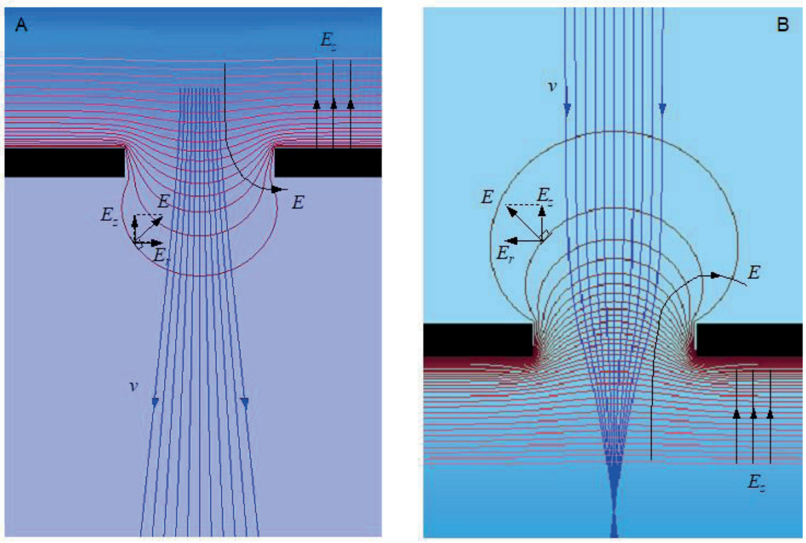
Problem b. (3 pts) Show that the focal point distance of a single-hole lens, where the electron leaves a decelerating field, expressed in the energy of the electron U outside the field, the strength of the field E and the diameter of the hole d, is given by:

$$f = \frac{4U}{eE}$$

Problem c. (2 pts) Determine the chromatic aberration of this lens, defined via the distance to the axis where an electron with energy U+ΔU and angle with the axis α lands in the focal plane:

$$\Delta r = C_c \cdot \frac{\Delta U}{U} \cdot \alpha$$

Problem d. (2 pts) Determine at which distance from the hole an electron is focused if it enters an accelerating field E with an energy U, as in figure B.



Necessary formula's:

Laplace equation in cylindric coordinates

$$\nabla^2\Phi = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}\right)\Phi = 0$$

Bessel equation

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(1 - \frac{n^2}{x^2}\right)y = 0$$

Solution to Bessel equation

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\,(k+n)!}\left(\frac{x}{2}\right)^{n+2k}$$

Prof. Dr. P. Kruit, MSc Y. Vos,
MSc M. W. H. Garming en MSc W. Zuidema

PION 2018 - Delft

White dwarfs, neutron stars and black holes

The end product of a star evolution is a compact object. Depending on the (initial) mass of the star, it becomes a white dwarf, a neutron star or a black hole. In the case of a white dwarf, the gravitational force compensates for the so called Fermi pressure. This is the pressure, exerted by a degenerated electron gas. For the central pressure in a white dwarf,

$$p_c \propto \frac{M^2}{R^4},$$

applies. Hydrostatic equilibrium is assumed. M is the mass of the white dwarf, and R is its radius. The equation of state is given by

$$p_c \propto \rho_c^\alpha.$$

α has the value of 5/3 when the material is non-relativistically degenerated, and 4/3 if it is degenerated relativistically; ρ is the density. If the electron gas is degenerated nonrelativistically, one can derive the mass-radius relation of a white dwarf; here we assume that the density is constant:

$$p_c \propto \rho_c^{5/3} \propto \left(\frac{M}{\frac{4}{3}\pi R^3}\right)^{5/3} \propto \frac{M^2}{R^4}.$$

This leads to:

$$R \propto M^{-1/3}.$$

This result shows that a heavier white dwarf is more compact.

Problem a. (2 pts) The more compact a white dwarf is, the stronger does the electron gas get degenerated. Show that that only one mass can satisfy the requirement of hydrostatic equilibrium in the case of a relativistically degenerated electron gas (the so called Chandrasekhar limit). Assume that the white dwarf has a constant density (homogeneous).

Problem b. (1 pt) If a white dwarf is situated in a dubble star system, mass transfer can occur when the accompanying star swells up to a giant. Explain why this scenario of a dubble star evolution can be used to clarify a supernova (type Ia).

Problem c. (3 pts) The Crab Pulsar is located in the middle of the Crab Nebula in the constellation Taurus and it shows a pulse time of 33 ms. Show that the Crab Pulsar is a neutron star ($M = M_\odot$ and R = 10 km) and not a white dwarf ($M = M_\odot$ and R = 10000 km). To prove this, derive an expression for the minimal rotational period of the object. (rotational speed equals the escape velocity $v = \sqrt{\frac{2GM}{R}}$)

Problem d. (2 pts) A black hole is a compact object whose escape velocity is higher than the speed of light. What is the size of the Schwarzschild radius (where the escape velocity is equal to that of light) for a black hole of $10\,M_\odot$?

Prof. Dr. L. Kaper

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