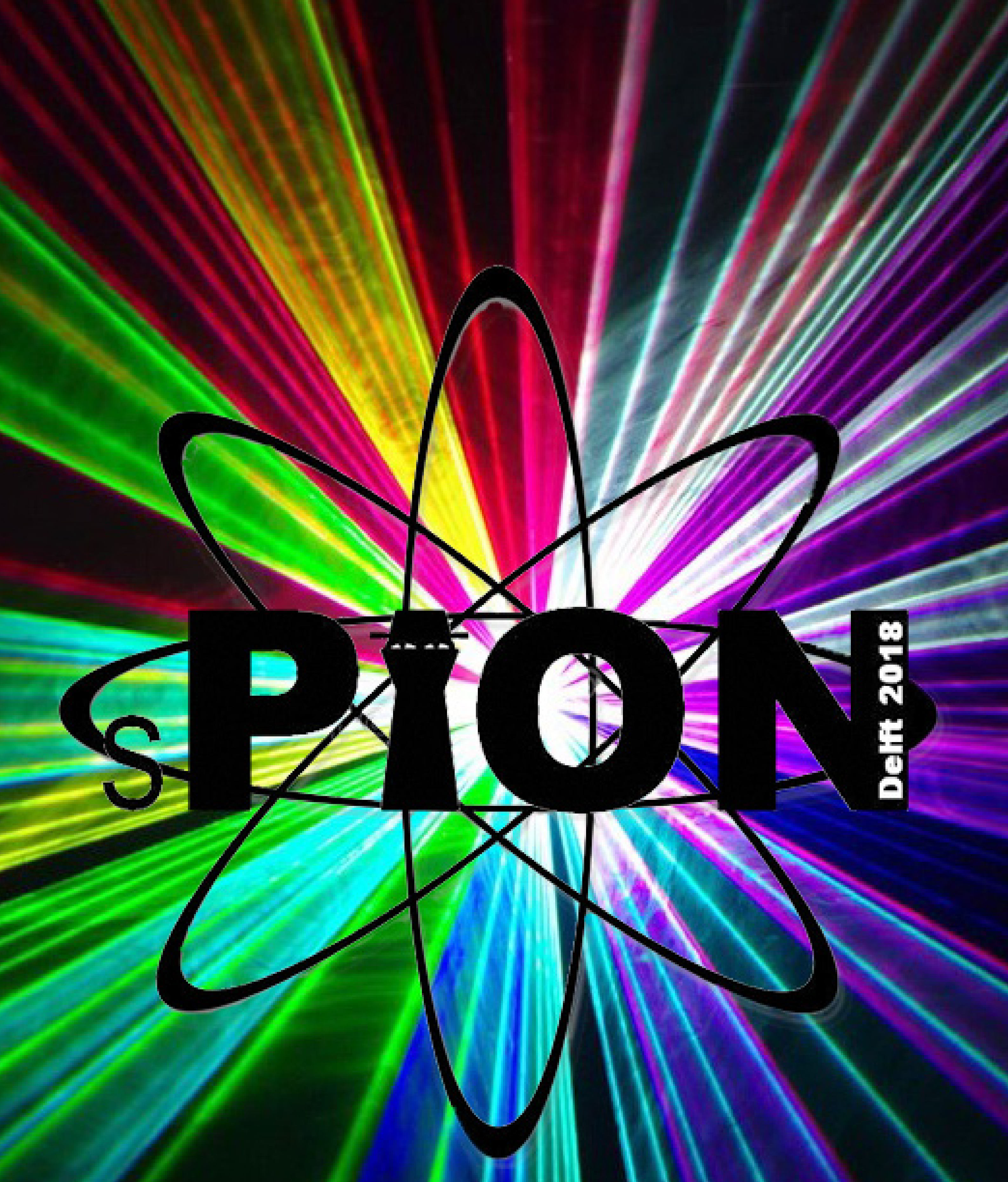

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PION 2018 - Delft

Introduction

Dear PION-competitor,

After a good cup of coffee or tea, an informative lecture and a filling lunch, it is time for the reason why you are here; the Olympiad! We would like to present the problems of PION 2018! With many thanks to the professors we have made a set of problems that combine many parts of the physical world surrounding you. These exercises will be more challenging than any exam you have ever made. We would like to wish you all the best with the Olympiad, show us why you should go to Zagreb this year!

Good luck!

Bart Warmerdam, Kim Bosch, Heike Smedes, Boyd Voet, Dirk van Bolhuis, Rixt Bosveld
PION commissie 2018

Rules and general information:

- Every problem should be made on a separate sheet.
- There will be 9 problems.
- Not every problem is worth the same number of points. The maximum of points that you can get per problem can be seen on the next page. With a total of 90 points.
- You have 3 hours to work on the problems.
- Write your team name and the name of the problem on each sheet.
- Only BINAS is allowed to use as reference.
- It is forbidden to communicate with any one but your teammates and the PION committee.
- It is forbidden to use a graphic calculator with more features than a TI-84 or equivalent calculator.

On the cover

The image on the cover is a laser show combined with an atom, which is the logo of the VVTP. Source: https://www.showandstage.de/Bundle-Laserworld-CS-2000RGB-MKII-Pangolin-Quickshow-10m-ILDA-Kabel-Case .

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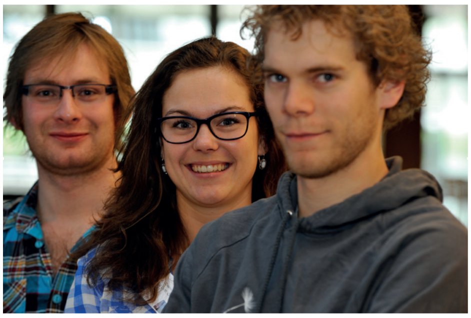 

Gezicht van de natuurkunde

    

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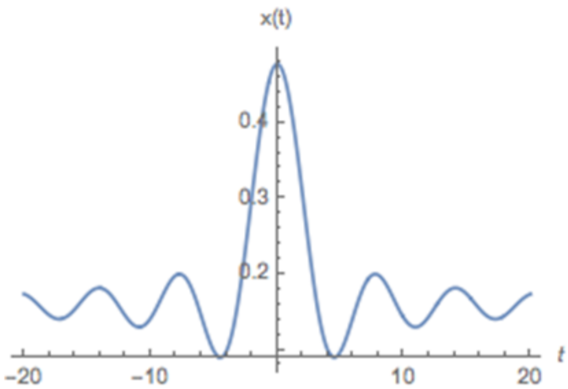
Holography

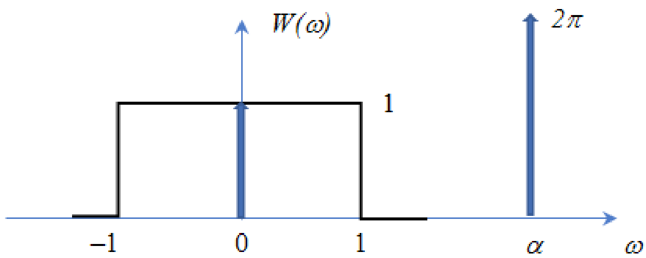
Problem a. (2 pts)

$$x(t) = \mathcal{F}^{-1}\{X(\omega)\} = \frac{1}{2\pi}\int_{-1}^{+1}(1+\delta(\omega))e^{j\omega t}\,d\omega$$
$$= \left(\frac{1}{2\pi}\int_{-1}^{+1} e^{j\omega t}\,d\omega\right)+\left(\frac{1}{2\pi}\right)=\frac{\sin(t)}{\pi t}+\left(\frac{1}{2\pi}\right)=\frac{\mathrm{sinc}(t)}{\pi}+\left(\frac{1}{2\pi}\right)$$



Problem b. (1 pt)

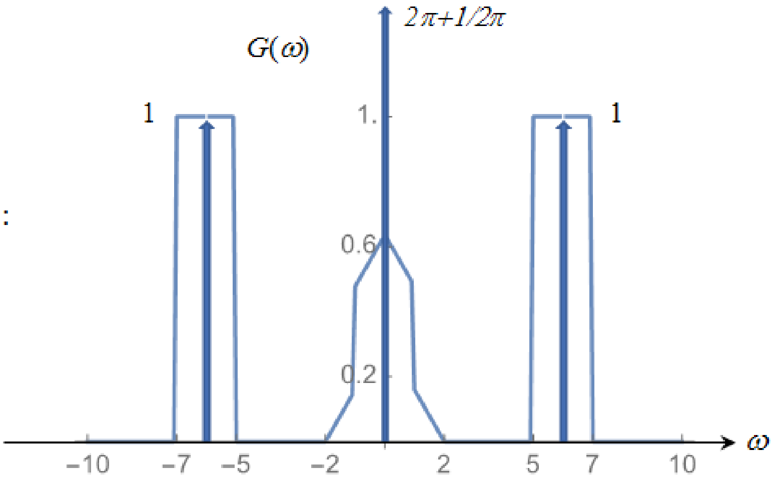
$$W(\omega) = X(\omega)+R(\omega) = X(\omega)+\mathcal{F}\{e^{j\alpha t}\}$$
$$= X(\omega)+2\pi\,\delta(\omega-\alpha)$$



Problem c. (2 pts)
X(ω) and δ(x) are real and even and δ(x-a) ⊗ δ(x-b) = δ(x-(a+b)) (⊗ = convolution). So we can write:

$$G(\omega) = \mathcal{F}\{|w(t)|^2\} = \mathcal{F}\{w(t)\,w^*(t)\} = \frac{1}{2\pi}W(\omega)\otimes W^*(-\omega)$$
$$= \frac{1}{2\pi}\big(X(\omega)+2\pi\,\delta(\omega-\alpha)\big)\otimes\big(X^*(-\omega)+2\pi\,\delta(-\omega-\alpha)\big)$$
$$= \frac{1}{2\pi}\big(X(\omega)\otimes X(\omega)\big)+X(\omega-\alpha)+X(\omega+\alpha)+2\pi\,\delta(\omega)$$

With $\alpha = 6$:



Problem d. (3 pts)
We can "demodulate" either the spectral term centered at ω = –α or the term at ω = +α to recover the original spectrum. Thus:

$$Y(\omega) = H(\omega)\mathcal{F}\{g(t)s(t)\} = H(\omega)\left(\frac{1}{2\pi}\right)\left(G(\omega)\otimes S(\omega)\right)$$

$$= H(\omega)\left(\frac{1}{2\pi}\right)\left(G(\omega)\otimes 2\pi\delta(\omega-\beta)\right) = H(\omega)G(\omega-\beta)$$

$\beta = +6$ or $\beta = -6$ (that is, $\pm\alpha$)

$\omega_c = 1$ (actually $1 < \omega_c < 4$ because of the distance between the spectral components)

$B = 1$

Problem e. (2 pts)
1) This imaging technique involves neither an aperture nor a lens in the way that a camera does. The film (or sensor) will be of finite spatial extent but that is not really the same as the aperture in a camera that hold the lens.

2) A camera, in general, works with whatever light is available. This imaging system (model) requires a monochromatic (single frequency α), coherent (single phase) light source for recording. The reconstruction must have the same frequency, $\beta = \alpha$. You might consider the effect in the reconstructed image that would be observed if the term s(t) had a fixed but unknown phase shift with respect to r(t).

3) The camera and this holographic imaging system will each have an exposure time so that is not special.

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The K.J-U Space Program

Problem a. (3 pts)
The distance from Earth to Alpha Centaurus (Binas table 32B) is 4.0*10^{16}m. Since all astronauts are males, human reproduction during the flight is not possible and the trip has to be completed within a human life time. Let's put this at 50 years = 1.6*10^9s. So the required velocity of the spaceship is 2.5*10^7m/s. This equals 0.08c, so relativistic effect scan be ignored (the mass of the traveling spaceship exceeds that of the spaceship at rest by a mere 0.3%). So the kinetic energy of the spaceship is

$$\frac{E_{kin}}{m} = \frac{1}{2}v^2 = 3.1 \cdot 10^{14}\ \text{J/kg}$$

This is much larger than its minimum escape energy, which can be calculated as

$$\frac{1}{2}mv^2 = \frac{GmM_{Earth}}{R_{Earth}} \rightarrow \frac{E_{esc}}{m} = \frac{1}{2}v^2 = \frac{GM_{Earth}}{R_{Earth}}$$

$$\xrightarrow{\text{Binas table 31}} \frac{E_{esc}}{m} = \frac{1}{2}\left(11200\frac{\text{m}}{\text{s}}\right)^2 = 6.3 \cdot 10^7\ \text{J/kg}$$

Problem b. (3 pts)
In table 58 of Binas we find the following binding energies:

C-H: $4.1 \cdot 10^5$ J/mol

C-C: $3.5 \cdot 10^5$ J/mol

O=O: $4.98 \cdot 10^5$ J/mol

C=O: $8.04 \cdot 10^5$ J/mol

O-H: $4.635 \cdot 10^5$ J/mol

So, during combustion, the released energy equals

$$E_{comb} = n \cdot 10^5 \left(2 \cdot 8.04 + 2 \cdot 4.635 - 3.5 - 2 \cdot 4.1 - \frac{3}{2} \cdot 4.98\right) = 6.2 \cdot n \cdot 10^5\ \text{J/mol}$$

(Compare the values given in table 56 of Binas:
$n=1$ (methane): $8.9 \cdot n \cdot 10^5$ J/mol
$n=2$ (ethabe): $7.8 \cdot n \cdot 10^5$ J/mol
$n=3$ (propane): $7.4 \cdot n \cdot 10^5$ J/mol
$n=4$ (butane): $7.2 \cdot n \cdot 10^5$ J/mol
So indeed, for large n we approach the value calculated with the simple model.)

The mass of 1 mole hydrocarbon equals

$$n(12.01 + 2 \cdot 1.008) = n \cdot 14.026\ \text{g/mol} = n \cdot 0.014026\ \text{kg/mol}$$

$$\rightarrow \frac{E_{comb}}{M_{FF}} = \frac{6.2 \cdot n \cdot 10^5\ \text{J/mol}}{n \cdot 0.014026\ \text{kg/mol}} = 4.4 \cdot 10^7\ \text{J/kg}$$

(If we use the Binas value for butane, we find
$$\frac{E_{comb}}{M_{FF}} = \frac{28.75 \cdot 10^5\ \text{J/mol}}{(4 \cdot 0.012 + 10 \cdot 0.001)\ \text{kg/mol}} = 5.0 \cdot 10^7\ \text{J/kg}$$)

Problem c. (1 pt)
The above leads to

$$\frac{M_{FF}}{m_{SS}} = \frac{3.1 \cdot 10^{14}}{4.4 \cdot 10^{7}} \approx 10^{7} \;\; !!!!$$

In which we have not taken into account that the mass M_{FB} of the fuel needs to be accelerated as well, which consequently leads to a much higher value of M_{FB} etc. etc.) In summary, president K.J-U's plan lacks a certain level of feasibility.

Problem d. (2 pts)
In a first approximation, the nuclear fusion reaction can be given by

$$2H_2 \rightarrow He$$

In this reaction , an amount of mass Δm= 4*1.008-4.003=0.0072 g/mol is transformed into energy through $E=mc^2$.

$$E_{nucl} = \Delta m \cdot c^2 = 0.000072 \frac{\text{kg}}{\text{mol}} \cdot \left(3 \cdot 10^8\right)^2 \frac{\text{J}}{\text{kg}} = 6.5 \cdot 10^{12} \frac{\text{J}}{\text{mol}} = \frac{6.5 \cdot 10^{12}}{0.004032} = 1.6 \cdot 10^{15} \frac{\text{J}}{\text{kg}}$$

This leads to

$$\frac{M_{H2}}{m_{SS}} = \frac{3.1 \cdot 10^{14}}{1.6 \cdot 10^{15}} \approx 0.2$$

Again, we have not taken into account that also the mass M_{H2} of the nuclear fuel needs to be accelerated, but in this case this leads to a relatively small correction in our calculation.

Particle Creation in Strong Fields

Problem a. (1 ptn)
The Hamiltonian

$$H = \frac{1}{2m}\mathrm{p}^2 + eE_z z$$

can be arbitrarily negative for z < 0 (if $eE_z > 0$ or $z > 0$ if $eE_z > 0$.). There is no groundstate.

Problem b. (2 pts)

$$z < 0 \ (\text{if } eE_z > 0 \text{ or } z > 0 \text{ if } eE_z > 0.)$$

Problem c. (2 pts)

$$V_{eff} = p_x^2c^2 + p_y^2c^2 + m^2c^4 - (\mathcal{E} - zeE_z)^2$$

The potential is an inverted parabola, with a well dened maximum of $V_{max} = p^2_x c^2 + p^2_y c^2 + m^2c^4$ *at* $z = E{=}e/E_z$.
The solution to $V_{eff}(z_{min}) = 0$ and $V_{eff}(z_{max}) = 0$ is

$$(\mathcal{E} - zeE_z)^2 = p_x^2c^2 + p_y^2c^2 + m^2c^4$$

$$eE_z z - \mathcal{E} = \pm\sqrt{p_x^2c^2 + p_y^2c^2 + m^2c^4}$$

$$z_{min,max} = \frac{\mathcal{E} \pm \sqrt{p_x^2c^2 + p_y^2c^2 + m^2c^4}}{(eE_z)}$$

Problem d. (1 pt)

$$V_{eff}(\mathcal{E}, e) = p_x^2c^2 + p_y^2c^2 + m^2c^4 - (\mathcal{E} - zeE_z)^2$$

Then

$$V_{eff}(-\mathcal{E}, -e) = p_x^2c^2 + p_y^2c^2 + m^2c^4 - (-\mathcal{E} + zeE_z)^2 = V_{eff}(\mathcal{E}, e)$$

$$V_{eff}(\mathcal{E}, -e)|_{z\to -z} = p_x^2c^2 + p_y^2c^2 + m^2c^4 - (\mathcal{E} - zeE_z)^2 = V_{eff}(\mathcal{E}, e)$$

Problem e. (1 pt)

$$p_z(z)^2c^2 + V_{eff}(z) = 0$$

Problem f. (1 pt)
From the answer in *e* it is obvious that *p(z)* is real whenever $V_{eff} < 0$. From the answer in c (V_{eff} is an inverted parabola) we know that this is the case for $z < z_{min}$ and $z > z_{max}$.

Problem g. (2 pts)
Careful: $p(z) = \frac{1}{c}\sqrt{-V_{eff}}$ is imaginary in this region as $V_{eff} > 0$. Thus $|p(z)| = \frac{1}{c}\left|\sqrt{V_{eff}}\sqrt{-1}\right| = \frac{1}{c}\sqrt{V_eff}$.

$$
\begin{aligned}
T &= \exp\left(-\frac{1}{\hbar}\int_{z_{min}}^{z_{max}} dz|p(z)|\right) = \exp\left(-\frac{1}{\hbar c}\int_{z_{min}}^{z_{max}} dz|\sqrt{V_{eff}}|\right) \\
&= \exp\left(-\frac{1}{\hbar c}\int_{z_{min}}^{z_{max}} dz\sqrt{p_x^2c^2 + p_y^2c^2 + m^2c^4 - (\mathcal{E} - zeE_z)^2}\right) \\
&= \exp\left(-\frac{1}{\hbar c E_z}\int_{\mathsf{z}_{min}}^{\mathsf{z}_{max}} d\mathsf{z}\sqrt{p_x^2c^2 + p_y^2c^2 + m^2c^4 - \mathsf{z}^2}\right) \\
&= \exp\left(-\frac{p_x^2c^2 + p_y^2c^2 + m^2c^4}{\hbar c E_z}\int_{-1}^{1} d\mathsf{z}\sqrt{1 - \mathsf{z}^2}\right) \\
&= \exp\left(-\frac{p_x^2c^2 + p_y^2c^2 + m^2c^4}{\hbar c E_z}\frac{\pi}{2}\right)
\end{aligned}
$$

Turtle vs. Hare

Problem a. (3 pts)
The speed of sound is u and we set it to 1, that is, we use units in which it is 1. It takes turtle 12/(/3/5) time units to reach the end of the track, i.e. 20 (time units). Next, she returns to her origin, which obviously takes another 20 time units. Thus in total: 40 time units. According to S, turtle's origin is then at position 40*(3/5) = 24 time units. Thus the beep needs to travel 24 time units and is heard by S at 40+24 = 64 time units.

For hare the analysis is quite the same: 12/(4/5) = 15 to reach the turning point and 30 time units to be back. Hare's origin is then 30*4/5 = 24 units away from S. The beep is heard after 30+24 = 54 time units. Clearly, hare wins (again).

Problem b. (3 pts)
Now, we are in the relativistic era! Hare has been training, but hasn't studied. He is so confident: hares always win! But turtle has both been training and following a course in classical mechanics and relativity. She understands that at high speeds, time is relative as well!

Turtle runs back and forth at 3/5c. In het reference frame, it takes 12/(3/5) = 20 light-time-units to reach the turning point and another 20 light-time-units back to the origin. Thus at (ctt,xt) = (40,0) turtle is back and pushes the light-flash button. For the referee in S, this event is:

$$ct = \frac{5}{4}\left(40 + \frac{3}{5}0\right) = 50$$

$$x = \frac{5}{4}\left(0 + \frac{3}{5}40\right) = 30$$

Thus the light flash is detected by the referee at ct = 80 light-time-units.
For Hare we get:
12/(4/5) = 15 -> (ct_h, x_h) = (30,0), which is in S:

$$ct = \frac{5}{3}\left(30 + \frac{4}{5}0\right) = 50$$

$$x = \frac{5}{3}\left(0 + \frac{4}{5}30\right) = 40$$

Thus the light flash is detected by the referee at ct = 90 light-time-units. Conclusion: Hare is the loser!

Problem c. (4 pts)
For any fixed value of L, we have that any given 'runner' will be hitting the light button at

$$ct' = 2\frac{L}{v/c}$$

This happens at x'=0, i.e. we have as event:

$$(ct', x') = \left(2\frac{L}{v/c}, 0\right)$$

Translated to the inertial frame of the referee:

$$ct = \gamma\left(2\frac{L}{v/c} + \frac{v}{c}0\right) = 2L\gamma\frac{1}{v/c}$$

$$x = \gamma\left(0 + \frac{v}{c}2\frac{L}{v/c}\right) = 2L\gamma$$

Consequently, the time of receiving the light flash is:

$$c\Delta t = 2L\gamma\left(1 + \frac{1}{v/c}\right)$$

Hence, this time is minimal if the function (set c=1 for simplicity, i.e. choose your units handy)

$$f(v) = \gamma(v)\left(1 + \frac{1}{v}\right)$$

It is easier to inspect f-squared and find the minimum of this function:

$$g(v) = \frac{\left(1 + \frac{1}{v}\right)^2}{1 - v^2} \rightarrow$$

$$g' = \frac{2(1+\frac{1}{v})\frac{-1}{v^2}(1-v^2) - (-2v)(1+\frac{1}{v})^2}{(1-v^2)^2} = \frac{2(1+\frac{1}{v})}{(1-v^2)^2}\left(\frac{-1}{v^2}(1-v^2) + v(1+\frac{1}{v})\right)$$

Minimum if:

$$\frac{-1}{v^2}(1-v^2) + v + 1 = 0 \rightarrow$$

$$v^3 + 2v^2 - 1 = 0 \rightarrow$$

$$\frac{1}{4}\left(-2v + \sqrt{5} - 1\right)(v+1)\left(2v + \sqrt{5} + 1\right) = 0$$

Since 0<v<1:

$$v = \frac{\sqrt{5} - 1}{2} \approx 0.618$$

For this value of v, the time for completing the run+traveling flash is shortest.

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Elasticity of Polymers

Problem a. (2 pts)

$< \boldsymbol{r}_i \cdot \boldsymbol{r}_j \geq 0$ for i≠j as all links are freely jointed, there is no directional preference.

$$\boldsymbol{R} = \boldsymbol{R}_N - \boldsymbol{R}_0 = (\boldsymbol{R}_N - \boldsymbol{R}_{N-1}) + (\boldsymbol{R}_{N-1} - \boldsymbol{R}_{N-2}) + \cdots + (\boldsymbol{R}_1 - \boldsymbol{R}_0)$$
$$= \sum_{i=0}^{N-1} \boldsymbol{r}_i$$

Thus: $< \boldsymbol{R}^2 > = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \boldsymbol{r}_i \cdot \boldsymbol{r}_j = Na^2$

Problem b. (3 pts)

$$P^E(\boldsymbol{k}) = \frac{1}{V} \int d^3R_0 \int d^3R_1 \ldots d^3R_{N-1} \frac{1}{(4\pi a^2)^N} \prod_{i-0}^{N-1} \delta(|\boldsymbol{r}_i| - a) e^{i\boldsymbol{kR}}$$

Where **R**=$\mathbf{R}_N$-$\mathbf{R}_0$

We transform the integration variables $\mathbf{R}_i$ **to** $\mathbf{r}_i$. This transform has Jacobian determinant 1, as can be seen from the transformation matrix.

1	0	..	0
-1	1	..	0
0	-1	1	0
...	...	-1	1
...	...	...	-1

This holds for the x, y and z components.

$$P_N^E(\boldsymbol{k}) = \frac{1}{(4\pi a^2)^N} \int d^3r_1 \ldots d^3r_{N-1} d^3r_0 e^{i\boldsymbol{k}(\boldsymbol{r}_0 + \cdots + \boldsymbol{r}_{N-1})} \prod_{i=0}^{N-1} \delta(|r_i| - a)$$
$$= \left(\frac{\sin(ka)}{ka}\right)^N$$

Sync function peaks at k=0.

$$lnP_N^E(\boldsymbol{k}) = N \cdot \ln\left(\frac{\sin(ka)}{ka}\right) = N \cdot \ln\left(\frac{ka - \frac{1}{b}(ka)^3}{ka}\right) \approx -\frac{N}{b}(ka)^2$$

Problem c. (3 pts)

$$P^E(\boldsymbol{R}) = \int e^{-\frac{N}{b}(ka)^2} e^{-i\boldsymbol{kr}} d^3k = \int e^{-\frac{Na^2}{b}\left(\boldsymbol{k} - \frac{3R}{iN}\right)^2} d^3k \, e^{-\frac{3R^2}{2Na^2}} = CZ(R)$$

Problem d. (2 pts)

Free energy $F = -kTln(Z) = Const + \frac{3k_bTR^2}{2Na^2}$

$$-\nabla_R F = force = -\frac{3k_bTR}{Na^2} \rightarrow Force = -k\boldsymbol{R}$$
$$\kappa = \frac{3k_bT}{Na^2}$$

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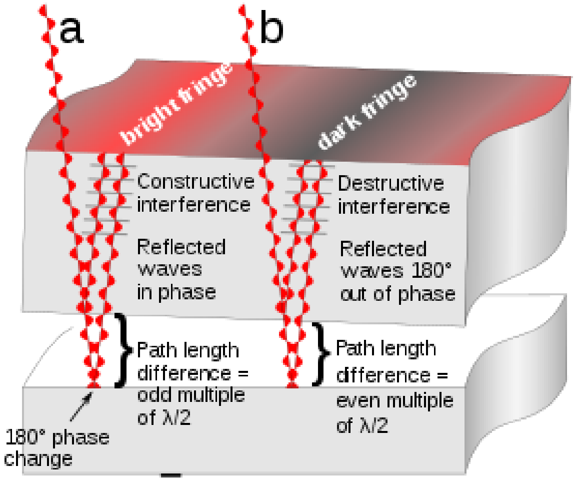
Newton Rings

Problem a. (0.5 pts)

Problem b. (0.5 pts)
These lines are created by interferences due to multiple reflection between the glass and the air around.

Problem c. (1.5 pts)



As discussed earlier, we see many reflection from the glass/air interference. We can neglect the fact that the glass is curved and believe that the reflection are collinear with in the incoming light (perpendicular to the air/glass interface) as seen in the drawing.

There is no light in the middle, cause we are looking for above. The two glass are touching, therefore there no reflection there, all the light cross the glass and we do not see reflection back from this location (there is light reflected by the others interface, but they are not relevant to our measurement because they are present at every position.

Below, we see the complementary image (bright in the center, and the inverted rings).

Problem d. (1 pt)
Using Pythagorus, we find that $OM^2 = OH^2 + HM^2$ so $OH^2 = R^2 - r^2$.

So $e(r) = r - OH = R - \sqrt{R^2 - r^2}$.

Problem e. (1 pts)
Using the approimation $R >> r$, we can write:

$$\sqrt{R^2 - r^2} = R\sqrt{1 - \left(\frac{r}{R}\right)^2} \approx R\left(1 - \frac{r^2}{2R^2}\right) = R - \frac{r^2}{2R}$$

So $e(r) \approx \frac{r^2}{2R}$

Problem f. (2 pts)
The total phase change is:

$$\delta = 2n_{air}e(r) + \frac{\lambda_0}{2} \approx n_{air}\frac{r^2}{2R} + \frac{\lambda_0}{2}$$

Counting also the reflection from the glass (the term $\lambda_0/2$).
So we have bright ring for $\delta = m\lambda_0$ so $r_{b,m}{}^2 \approx (m-1/2)\lambda R$ with $\lambda = \lambda_0/n_{air}$.

Problem g. (0.5 pts)
The dark rings appear for $\delta = (m + 1/2)\lambda_0$ so $r_{d,m}{}^2 \approx m\lambda R$ with $\lambda = \lambda_0/n_{air}$.

Problem h. (0.5 pts)
The first dark ring occurs at:

$$\frac{\lambda}{2} = \frac{\lambda_0}{2n_{air}} = \frac{0.570\ \mu m}{2} = 0.285\mu m$$

An inch is 25400µm so 25400µm/88850 = 0.28587µm. Amazing result!

Problem i. (0.5 pts)
You see that the formula find in 7 is indeed linearly depending linearly on m, for constant color. This is what he called "arithmetical progression". So It is correct.

Problem j. (1 pt)
Two consecutives dark rings verify:

$$r_{d,m+1}^2 - r_{d,m}^2 = (m+1)\lambda R - m\lambda R = \lambda R$$

But also

$$\Delta r_{d,m} = r_{d,m+1} - r_{d,m} = \sqrt{(m+1)\lambda R} - \sqrt{m\lambda R} \approx \frac{1}{\sqrt{m+1} + \sqrt{m}}\sqrt{\lambda R}$$

$\Delta r_{d,m}$ decreases when m increases. When m is very large we get:

$$\Delta r_{d,m} \approx \sqrt{\frac{\lambda R}{m}}$$

Problem k. (0.5 pts)
As seen in the equation the bright light position is depending on the wavelength. The inner radius should see the low wavelength first (blue) and the higher wavelength further (red).

Problem l. (1 pt)

$$r_{d,m}{}^2 \approx m\lambda R = \frac{m\lambda_0 R}{n_{air}}$$

The last dark ring should appear for $r_{s,max} \approx \frac{\Phi}{2}$ so

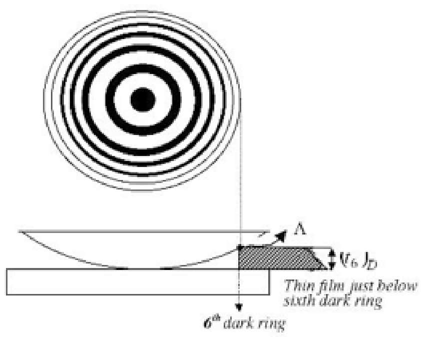
$$m_{max} \approx round\left(n_{air}\frac{\left(\frac{\Phi}{2}\right)^2}{\lambda_0 R}\right) = round(33.95) = 33$$

We observe 33 dark rings in total.

If we observer 45 dark rings to oppose to 33 before, the ratio of index of refractions should be 45/33=1.36
A reasonable value

Problem m. (0.5 pts)

We can use these rings to measure the curvature of a Lens. We can also use it to measure the thickness of object. See below

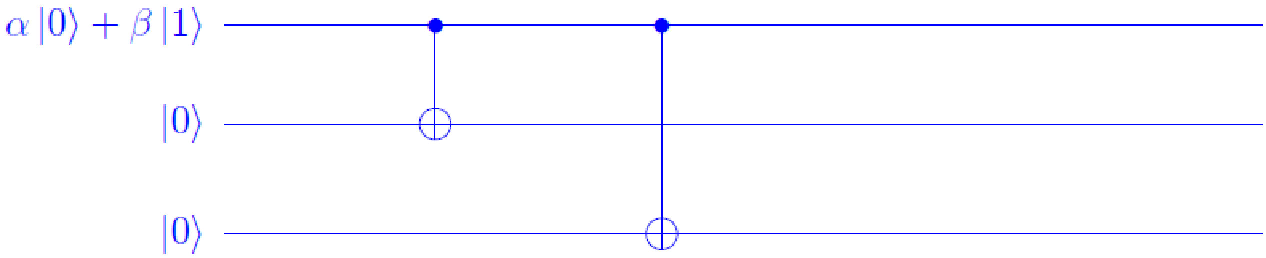


Problem n. (0.5 pts)
If we have a mirror instead of the glass plate, we do not observe anything anymore, because the mirror reflects everything, and not interference can be seen.

Problem o. (0.5 pts)
Here we see the rings coming from a not well polished lens. This technique helps us to check the quality of the fabrication of a lens.

Correcting Quantum Errors

Problem a. (1 pt)

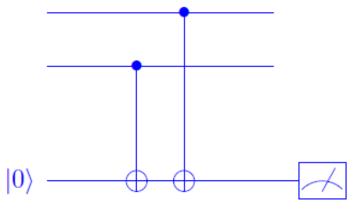


Problem b. (2 pts)
This is an entangled state, as one cannot write this state as

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle$$

Furthermore, measurement of one qubit also collapses the other qubits.

Problem c. (4 pts)
When looking at the error correcting circuits, one can recognize the following subcircuit, that is repeated twice (on dierent qubits),



Error	Measurement outcome Ancilla 1	Measurement outcome Ancilla 2
No error	0	0
X_1	1	0
X_2	1	1
X_3	0	1

At the start of the circuit, we assume to have the following wavefunction,

$$|\psi\rangle = (\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle) \otimes |0\rangle_{ancilla}$$

After executing the gates of the circuit, the wavefunction will evolve to:

$$|\psi\rangle = (\alpha|000\rangle + \delta|110\rangle) + (\beta|011\rangle + \gamma|101\rangle)$$

When a projective measurement is performed on the ancilla qubit, the state of the top two qubits will either be (neglecting normalization),

$$\alpha|00\rangle + \delta|11\rangle$$

if the measurement returns 0 or

$$\beta |01\rangle + \gamma |10\rangle$$

when the measurement returns 1. Thus, the circuit returns 0 if the parity of the top two qubits is even and 1 if the parity is uneven.

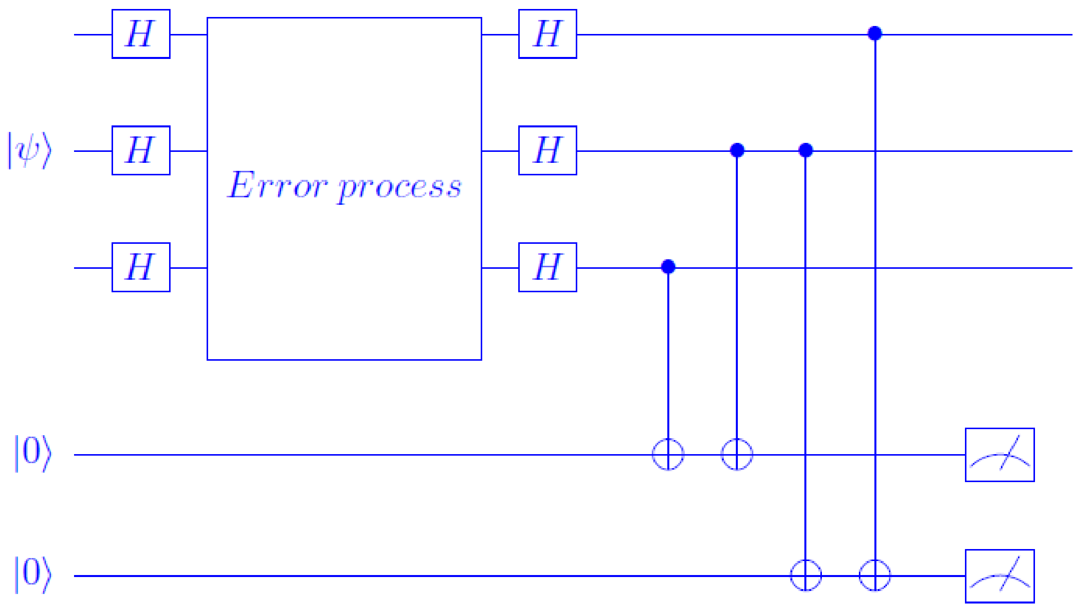
Returning to the original circuit. The function of the ancilla qubits is to detect the parity of the wavefunction. When an X error occurs, the symmetry of the wavefunction will change. The ancilla qubits will be able to detect this as a change in parity. Combining the two parity measurements, we can detect whether an X error occurred and also on which qubit it occurred. For example, an X error on qubit one would change the wavefunction to:

$$|\psi\rangle = \alpha |100\rangle + \beta |011\rangle$$

Where the parity of the rst two qubits is uneven and the parity between the last two even. The full truth table is given in table 1. We here assumed that the likelihood of having an error on each of the qubits is the same and that the error rate is low enough that it is unlikely to have multiple errors at the same time.

Problem d. (1 pt)
The correction of the errors is quite straightforward, as the measurement circuit determines on which qubit the error has occurred. You can simply perform an X operation on the qubit where the error occurred.

Problem e. (2 pts)
We can convert Z errors into X errors via a basis transformation that maps $|0\rangle$ onto $(|0\rangle + |1\rangle)/\sqrt{2}$ and $|1\rangle$ onto$(|0\rangle - |1\rangle)/\sqrt{2}$. To implement this, we add Hadamard operations (H) before and after the error process, as indicated in the circuit below.



Electrostatic Lenses in the MAPPER Electron Lithography Machine

Problem a. (4 pts)
Laplace equation in cylindrical coordinates

$$\nabla^2\Phi = \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}\right)\Phi = 0$$

Use separation of variables, assume:

$$\Phi(r,\phi,z) = R(r)\psi(\phi)Z(z)$$

Fill into Laplace eqn:

$$\psi Z\, R''(r) + \frac{1}{r}\,\psi Z\, R'(r) + \frac{1}{r^2} RZ\,\psi''(\phi) + R\psi\, Z''(z) = 0$$

Separate the variables:

$$\frac{R''(r)}{R} + \frac{1}{r}\frac{R'(r)}{R} + \frac{1}{r^2}\frac{\psi'(\phi)}{\psi} + \frac{Z''(z)}{Z} = 0$$

This means that $\frac{Z''(z)}{Z}$ is equal to some constant, that we will call p^2. As the problem is rotationally symmetric, the term containing $\psi'(\phi)=0$ and drops out.

Now, we move to a different variable, namely $\rho=pr$.
This means that

$$R''(r) = \frac{d^2R}{dr^2} = \frac{d^2R}{d\rho^2}\frac{d^2\rho}{dr^2} = R''(\rho)\cdot p^2$$

and

$$R'(r) = R'(\rho)\cdot p$$

Filling yields:

$$R''(\rho) + \frac{1}{\rho}\,R'(\rho) + R = 0$$

This is the Bessel differential equation with n=0. The solution is

$$R_{n=0}(pr) = J_{n=0}(pr) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{pr}{2}\right)^{2k}$$

Now the total solution for the potential Φ becomes

$$\Phi(r,\phi,z) = R(r)\psi(\phi)Z(z)$$
$$= \psi(\phi)Z(z)\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{pr}{2}\right)^{2k}$$

As $Z^{(k)}(z)=p^k Z$, this can be rewritten to

$$\begin{aligned}\Phi(r,\phi,z) &= \psi(\phi)Z(z)\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{pr}{2}\right)^{2k} \\ &= \psi(\phi)Z(z)\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{r}{2}\right)^{2k} p^{2k} \\ &= \psi(\phi)\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{r}{2}\right)^{2k} Z^{(2k)}(z)\end{aligned}$$

As we want to express the answer in terms of Φ(0,z), we want to rewrite Z and Φ in terms of this.

$$\Phi(r=0,z) = \frac{1}{2}\,\psi(\phi)Z(z) \rightarrow \psi(\phi)Z(z) = 2\Phi(r=0,z)$$

Thus:

$$\Phi(r,z) = 2\sum_{k=0}^{\infty}\frac{(-1)^k}{(k!)^2}\left(\frac{r}{2}\right)^{2k}\frac{\partial^{2k}\Phi(0,z)}{\partial z^{2k}}$$

Expand first 3 terms:

$$\Phi(r,z) = \Phi(0,z) - \frac{r^2}{4}\frac{\partial^2\Phi(0,z)}{\partial z^2} + \frac{r^4}{64}\frac{\partial^4\Phi(0,z)}{\partial z^4} - \cdots$$

To get the fields, differentiate the potential

$$E_z(r,z) = -\frac{\partial\Phi(r,z)}{\partial z} = -\Phi'(z) + \frac{r^2}{4}\Phi^{(3)}(z) + \cdots = E_z(0,z) - \frac{r^2}{2}E_z''(0,z) + \cdots$$

$$E_r(r,z) = -\frac{\partial\Phi(r,z)}{\partial r} = \frac{r}{2}\Phi'' - \frac{r^3}{16}\Phi^{(4)}(z) + \cdots = -\frac{r}{2}E_z'(0,z) + \frac{r^3}{16}E_z^{(3)}(0,z) + \cdots$$

The lens working is the first term, the spherical aberrations are the second term.

Problem b. (3 pts)
For the derivation of the of the focal length we assume that the change of angle is relatively small. The lens effect can then be approximated by the radial change in direction of the electrons:

$$\Delta(mv_r) = \Delta p_r = \int eE_r dt = \int \frac{eE_r}{v_z}dz$$

With the equation for Er derived above, $E_r = -\frac{r}{2}\frac{dE_z}{dz}$, we find

$$E_r dz = -\frac{r}{2}dE_z$$

Substituting this into the first equation gives us:

$$-\int \frac{er}{2v_z}dE_z$$

We integrate this equation from the entrance of the lens where the field strength is E to where the lens effect has died out E=0. We assume v_z to be constant as the change of angle is relatively small. We obtain:

$$\Delta p_r = \frac{er}{2v_z}E \rightarrow \alpha = \frac{\Delta v_r}{v_z} = \frac{r}{f} = \frac{erE}{2mv_z^2}$$

Thus

$$f = \frac{4U}{eE}$$

Problem c. (2 pts)
We want an expression for:

$$\boldsymbol{\Delta r = C_c \frac{\Delta U}{U}\ \alpha}$$

Chromatic aberrations are described as a spread in focal distance due to an energy spread. Hence, we can remain in the paraxial limit.

$$r = \alpha f \rightarrow \Delta r = \alpha \Delta f$$

(Make a drawing to clearly illustrate this)
Based on the answer from b):

$$\Delta f = \frac{4\Delta U}{E} \rightarrow \frac{\Delta f}{f} = \frac{\Delta U}{U}$$

Combine these 3 equations to obtain the C_c term:

$$\boldsymbol{\Delta r = C_c \frac{\Delta U}{U}\ \alpha = \alpha \Delta f = \alpha \frac{\Delta \mathrm{f}}{\mathrm{f}}\ f = f \frac{\Delta U}{U}\ \alpha}$$

Hence, we can conclude that :

$$C_c = f$$

Problem d. (2 pts)
Unlike b), the electron experiences an radial inward force first, and next gets accelerated by the electric field. We know v_r after the initial lens effect:

$$v_r = \frac{er}{2mv_z}E$$

From this we can calculate the time that the beam crosses the optical axis:

$$t = \frac{r}{v_r} = \frac{2mv_z}{eE} = \frac{2^{\frac{3}{2}}\sqrt{mU}}{eE}$$

After the initial lens effects, the electron gets accelerated by the electric field. We know from a parallel plate capacitor that the electric field is constant. Therefore the acceleration is constant. The change in focal point becomes:

$$\Delta f = \int\int \frac{eE_z}{m} dt^2 = \frac{eE}{2m} t^2$$

Combine with the time calculated previously:

$$\Delta f = \frac{eE}{2m} \frac{8mU}{e^2E^2} = \frac{4U}{eE} \to f = f_0 + \Delta f = \frac{4U}{eE} + \frac{4U}{eE} = \frac{8U}{eE}$$

White dwarfs, neutron stars and black holes

Problem a. (2 pts)
Since it is homogeneous,

$$p \propto \rho_c \propto M/R^3$$

HE:

$$p_c \propto \frac{M^2}{R^4} \propto \left(\frac{M}{R^3}\right)^{4/3} \propto \frac{M^{4/3}}{R^4}$$

So M is constant.

Problem b. (1 pt)
Mass is transferred to energy.

Problem c. (3 ptn)
p_s=33 ms. Minimal rotational period:

$$\frac{mv^2}{R} = \frac{GMm}{R^2}$$

with $v = \Omega R$ and $\Omega = \frac{2\pi}{p}$. So

$$\Omega^2 R = \frac{GM}{R^2}$$

$$\frac{4\pi^2}{p^2} = \frac{GM}{R^3}$$

$$P_{min} = \sqrt{\frac{4\pi^2 R^3}{GM}}$$

White dwarf:

$$P_{min} = 17.2s > 33ms$$

Neutron star:

$$P_{min} = 5.4 \cdot 10^{-4}s < 33ms$$

Problem d. (2 ptn)

$$\frac{1}{2}mv^2 > \frac{GMm}{R}$$

$$R_s = \frac{2GM}{c^2} \simeq 30km$$

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